nents of the subtrahend are simply stricken from the minuend, and the result of this cancellation is the difference required, *i. e.*, $87-52=35$.

If characters like those proposed were in use, it seems probable that children could learn addition tables with far greater ease and certainty than is the case at present.

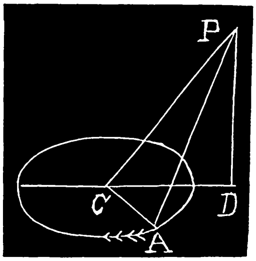
Finally, it may be remarked that if addition be carried out in the ordinary manner, these characters will be quite as serviceable as are the Arabic numerals, and being adapted to the mode of addition described above, a new and convenient way of proving the work will be afforded.

THE CONVEX SURFACE OF AN OBLIQUE CONE.

By F. P. MATZ, Reading, Pa.

Legendre, in his *Théorie des fonctions elliptiques*, has devised a method for finding the convex surface of an oblique cone having a circular base.

We propose the following method: Let $CA=r$, $PD=a$, and $\angle PCD=\omega$. Represent the uniformly varying $\angle DCA$ by θ; then since $CD=a\cot\omega$, we have by trigonometry,



$$\cos\theta=\frac{a^2\cot^2\omega+r^2-(AD)^2}{2ar\cot\omega}\text{.........}(1).$$

$$\therefore AD=\surd(a^2\cot^2\omega+r^2-2ar\cot\omega\cos\theta)\text{.........}(2),$$

and $AP=\surd(a^2\operatorname{cosec}^2\omega+r^2-2ar\cot\omega\cos\theta)$ .... (3),

which *line* represents an element of convex surface of the oblique cone.

$$\therefore S=2r\int_0^\pi\surd(a^2\operatorname{cosec}^2\omega+r^2-2ar\cot\omega\cos\theta)d\theta\text{..........}(4).$$

Transforming (4) under the supposition that $\theta=\pi+2\phi$, we get

$$S=2r\int_0^{\frac{1}{2}\pi}\surd(a^2\operatorname{cosec}^2\omega+r^2+2ar\cot\omega\cos2\phi)\times2d\phi\text{.........}(5),$$

$$=4r\int_0^{\frac{1}{2}\pi}\surd(a^2\operatorname{cosec}^2\omega+r^2+2ar\cot\omega-2ar\cot\omega\sin^2\phi)d\phi\text{..........}(6),$$

$$=4r\int_\theta^{\frac{1}{2}\pi}\sqrt{\left[\left(\frac{a^2+2ar\sin\omega\cos\omega+r^2\sin^2\omega}{\sin^2\omega}\right)-\left(\frac{2ar\sin\omega\cos\omega}{\sin^2\omega}\right)\sin^2\phi\right]}d\phi\text{..........}(7).$$

Representing *in order* these parenthetical expressions by m^2 and n^2, (7) gives

$$S=4mr\int_0^{\frac{1}{2}\pi}\sqrt{[1-(n^2/m^2)\sin^2\phi]}d\phi \text{..........}(8);$$

that is, according to Legendre's system of notation for elliptic functions,

$$n^2/m^2=k^2=\frac{2ar\sin\omega\cos\omega}{a^2+2ar\sin\omega\cos\omega+r^2\sin^2\omega}\text{..........}(9).$$

By means of (9), we have from (8),

$$S=4mr\int_0^{\frac{1}{2}\pi}\sqrt{(1-k^2\sin^2\phi)}d\phi=4mrE(k,\tfrac{1}{2}\pi)$$

$$=2\pi mr\left[1-\sum_{p=1}^{p=\infty}\left(\frac{1.3.5\text{.........}(2p-1)}{2.4.6\text{..................}2p}\right)^2\left(\frac{k^{2p}}{2p-1}\right)\right]\text{.........}(10),$$

which is the formula required.

Corollary. If $\omega=90°$, $k^2=0$; and under this supposition, we have from (8),

$$S=4r\sqrt{(r^2+a^2)}\int_0^{\frac{1}{2}\pi}d\phi=2\pi r\sqrt{(r^2+a^2)},$$

which is the formula for the convex surface of a right circular cone.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problem 99. March, 1899; March, 1900; April, 1904.

Solution by F. H. SAFFORD, Ph. D., University of Pennsylvania.

In the "seven" problem the writer has used the following method to show that there is a single solution, notation apart. Let the natural order *1234567* be contained in every solotion. There are fifteen arrangements containing *124* which do not conflict with the natural order. The same is true for *125*, and for *126*, while there are seventeen for *127*. Advancing the figures by two units, the arrangements containing *346*, *348*, *341*, *342* are obtained; similarly for *561*, etc., then for *713*, etc., ending with the sets containing *672*, *673*, *674*, *675*. In this